

chapter 13

a. stage 1: separate total variability (SS and df) into two basic components: between treatments and within treatments

stage 2: separate the between variability into 3 components that will form the numerators for the 3 F-ratios: variance due to factor A, variance due to factor B and variance due to the interaction

4.

		factor B: coffee		
		Decaf	Regular	
Factor	0-day	$M=4$	$M=8$	Overall $M=6$
A: delay	10-day	$M=2$	$M=6$	Overall $M=4$
		Overall $M=3$	Overall $M=7$	

c) factor A has 2 levels: 0-day and 10-day delay

b) No interaction, there is a constant 2 point difference between 0-day and 10-day that does not depend on the level of factor B

c. $n=10$

		Factor B			
		B_1	B_2	B_3	
a)	A_1	$M=8$	$M=12$	$M=28$	$A_1 \text{ mean} = 16$
	A_2	$M=4$	$M=8$	$M=?$	$A_2 \text{ mean} = 12$
		$B_1 = 6$	$B_2 = 10$	B_3	

difference between A_2 and A_1 in B_1 and $B_2 = 4$, therefore this should be maintained in B_3 $28 - 4 = 24$
 $M = 24$

b) For there to be no interaction, the mean differences have to be different for both factors. If $M = 24$
 $B_3 \text{ mean} = 26$, creating a difference of 12 between B_1, B_2 and itself hence no interaction with A which has a 4 point difference

8. Factor A - 2 levels
 Factor B - 3 levels
 $n=10$

- df for factor A = $2 - 1 = 1$
- df for factor B = $3 - 1 = 2$
- df for interaction = $2 - 1 = 1$

$$10, \quad n=4 \quad k=0.25$$

Total Variability

$$SS_{\text{total}} = \sum X^2 - \frac{G^2}{N} = 5820 - \frac{336^2}{24} = 1116$$

$$df_{\text{total}} = N - 1 = 24 - 1 = 23$$

Within treatments variability

$$df_{\text{within}} = \sum df_{\text{each element}} = (3 \times 6) = 18$$

$$SS_{\text{within treatments}} = 48 + 42 + 66 + 72 + 54 + 42 = 324$$

$$SS_{\text{between}} = SS_{\text{total}} - SS_{\text{within}} = 1116 - 324 = 792$$

$$MS_A = \frac{SS_A}{df_A}$$

$$df_A = \text{number of rows} - 1 = 2 - 1 = 1$$

$$MS_B = \frac{SS_B}{df_B}$$

$$df_B = \text{number of columns} - 1 = 3 - 1 = 2$$

$$MS_{A \times B} = \frac{SS_{A \times B}}{df_{A \times B}}$$

$$df_{A \times B} = df_{\text{between}} - df_A - df_B = 5 - 1 - 2 = 2$$

$$T_{B1} = 88$$

$$T_{A1} = 204$$

$$T_{B2} = 88$$

$$T_{A2} = 132$$

$$T_{B3} = 160$$

$$SS_A = \sum \frac{T_{\text{rows}}^2}{n_{\text{row}}} - \frac{G^2}{N}$$

$$= \frac{204^2}{12} + \frac{132^2}{12} - \frac{336^2}{24} = 216$$

$$SS_B = \frac{88^2}{8} + \frac{88^2}{8} + \frac{160^2}{8} - \frac{336^2}{24} = 432$$

$$MS_A = \frac{216}{1} = 216$$

$$MS_B = \frac{432}{2} = 216$$

$$MS_{A \times B} = \frac{216}{1} = 216$$

$$F_A = \frac{MSA}{MS \text{ within}}$$

$$MS \text{ within} = \frac{SS \text{ within}}{df \text{ within}} = \frac{324}{18} = 18$$

$$F_A = \frac{216}{18} = 12$$

$$F_B = \frac{216}{18} = 12$$

$$F_{A \times B} = \frac{216}{18} = 12$$

Source	SS	df	MS
Between treatments		1	
Factor A	48	1	48
Factor B	48	2	24
A x B interaction	12	1	12
Within treatments	144	24	6
Total		29	

$F = \frac{2.00}{1.00} = 2.00$
 $F = \frac{4.00}{2.00} = 2.00$
 $F = \frac{2.00}{1.00} = 2.00$

$$df_A = k-1 = 2-1 = 1$$

$$df_B = k-1 = 3-1 = 2$$

$$n=5$$

$$df_{A \times B} = \text{df of all factors} = (2-1) \times (3-1) = 2 \times 1 = 2$$

$$df \text{ within} = 24$$

$$df \text{ total} = N-1 = 30-1 = 29$$

$$MS \text{ within} = \frac{SS \text{ within}}{df \text{ within}} = \frac{144}{24} = 6$$

14.

$S_1^2 = 13$	$T_1 = 50$
$S_2^2 = 11$	$T_2 = 20$
$S_3^2 = 11$	$T_2 = 60$
$S_3^2 = 13$	$T_4 = 70$

$$df \text{ total} = 20-1 = 19$$

$$df \text{ within} = 20-4 = 16$$

$$df \text{ between} = 3$$

$$SS \text{ total} = 472$$

$$SS \text{ within} = 192$$

$$SS \text{ between} = 280$$

$$df_A = 1$$

$$df_B = 1$$

$$df_{A \times B} = df \text{ between} - df_A - df_B = 3-1-1 = 1$$

$$T_{B1} = 110$$

$$T_{B2} = 90$$

$$T_{A1} = 70$$

$$T_{A2} = 130$$

$$SS_A = \sum \frac{T_{2 \text{ rows}}^2}{n \text{ rows}} - \frac{G^2}{N}$$

$$= \frac{70^2}{5} + \frac{130^2}{5} - \frac{200^2}{20} = 2040$$

$$SS_B = \sum \frac{T_{2 \text{ col}}^2}{n \text{ col}} - \frac{G^2}{N}$$

$$= \frac{110^2}{10} + \frac{90^2}{10} - \frac{200^2}{20} = 2040$$

$$MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{192}{16} = 12$$

$$F_A = \frac{MS_A}{MS_{within}} = \frac{2360}{12} = 196.67$$

$$F_B = \frac{MS_B}{MS_{within}} = \frac{2040}{12} = 170$$

Because the F values are different there is main effects.

$$F_{AB} = \frac{MS_{AB}}{MS_{within}} \Rightarrow MS_{AB} = \frac{SS_{AB}}{df_{AB}} = \frac{2360}{1} = 2360$$

$$= \frac{93 \cdot 33}{12} = 7.775$$

$$F_{crit} = 3.23887$$

b) If $\alpha = 0.01$

only the critical value changes to 5.4119

$$16) \quad T_1 = 42 \quad T_2 = 108 \\ T_2 = 108 \quad T_4 = 108$$

$$\bar{X} = 7 \\ \bar{X} = 18$$

$$\bar{X} = 18 \\ \bar{X} = 17$$

$$S_1^2 = 16 \\ S_2^2 = 16$$

$$S_3^2 = 14 \\ S_4^2 = 18$$

$$SS_{between} = 516$$

$$SS_{within} = 320$$

$$SS_{total} = 836$$

$$df_{within} = 5 \times 4 = 20$$

$$df_{between} = k - 1 = 4 - 1 = 3$$

$$MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{320}{20} = 16$$

$$MS_{between} = 172$$

$$df_A = Rows - 1 = 2 - 1 = 1$$

$$df_B = Columns - 1 = 2 - 1 = 1$$

$$\begin{array}{cc} \text{Rows} & \text{Columns} \\ T = 150 & 150 \\ T = 210 & 210 \end{array}$$

$$n = 12$$

$$G = 720360 \\ N = 24$$

$$SSA = \frac{150^2}{12} + \frac{210^2}{12} - \frac{360^2}{24} = 150$$

$$SS_B = \frac{150^2}{12} + \frac{210^2}{12} - \frac{360^2}{24} = 150$$

$$MS_A = \frac{SS_A}{df_A} = \frac{150}{1} = 150$$

$$F_A = \frac{MS_A}{MS_{within}} = \frac{150}{16} = 9.375$$

$$F_B = \frac{MS_B}{MS_{within}} = \frac{150}{16} = 9.375$$

$$MS_{A \times B} = 9.375$$

$$F_{crit} = 3.0983$$

$$F = 10.75 = \frac{MS_{between}}{MS_{within}} = \frac{172}{16}$$

We reject the null hypothesis and conclude that there is interaction

18.

Source	SS	df	MS	
Between treatments	224	7	32	
Factor A	16	1	16	$F = 8.00$
Factor B	170	3	56.67	$F = 28.335$
A x B interaction	38	3	12.67	$F = 6.33$
Within treatments	144	72	2	
Total	368	86		

$$S_{total} = S_{within} + S_{between} = 224 + 144 = 368$$

$$df_{within} = 72 = (n-1)$$

$$df_{between} = 8-1 = 7$$

$$df_{A \times B} = df_{between} - df_A - df_B = 7 - 3 - 1 = 3$$

$$MS_A = \frac{SS_A}{df_A} = \frac{16}{1}$$

$$MS_B = \frac{SS_B}{df_B} = \frac{170}{3} = 56.67$$

$$MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{144}{72} = 2$$

$$MS_{between} = \frac{SS_{between}}{df_{between}} = \frac{224}{7} = 32$$

$$MS_{A \times B} = \frac{SS_{A \times B}}{df_{A \times B}} = \frac{38}{3} = 12.67$$

$$F_A = \frac{MS_A}{MS_{within}} = \frac{16}{2} = 8.00$$

$$F_B = \frac{MS_B}{MS_{within}} = \frac{56.67}{2} = 28.335$$

$$F_{A \times B} = \frac{MS_{A \times B}}{MS_{within}} = \frac{12.67}{2} = 6.33$$

20. a) Total variability $SS_{total} = \sum x^2 - \frac{G^2}{N}$

$$G = (24 + 72 + 76 + 36) = 208$$

$$N = 16$$

$$n = 4$$

$$SS_{total} = (198 + 1368 + 1492 + 390) - \frac{208^2}{16} = 744$$

$$T_{col} = 100$$

$$T_{row} = 108$$

$$T_{row} = 96$$

$$T_{row} = 112$$

$$df_{total} = N - 1 = 16 - 1 = 15$$

$$SS_{within\ treatments} = \sum SS\ each\ element = 54 + 72 + 48 + 66 = 240$$

$$df_{within\ treatments} = 3 + 3 + 3 + 3 = 12$$

$$SS_{between\ treatments} = SS_{total} - SS_{within} = 744 - 240 = 504$$

$$df_A = 1$$

$$df_B = 1$$

$$df_{A \times B} = df_{between} = df_A - df_B = (4 - 1) - 1 - 1 = 1$$

$$SS_A = \sum \frac{T^2}{n} - \frac{G^2}{N} = \left(\frac{96^2}{4} + \frac{112^2}{4} \right) - \frac{208^2}{16} = 2736$$

$$SS_B = \frac{100^2}{4} + \frac{108^2}{4} - \frac{208^2}{16} = 2712$$

$$MS_A = \frac{SS_A}{df_A} = \frac{2736}{1} = 2736$$

$$MS_B = \frac{SS_B}{df_B} = \frac{2712}{1} = 2712$$

$$MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{240}{12} = 20$$

$$MS_{between} = \frac{SS_{between}}{df_{between}} = \frac{504}{3} = 168$$

$$F_A = \frac{MS_A}{MS_{within}} = \frac{2736}{20} = 136.8$$

$$F_B = \frac{MS_B}{MS_{between}} = \frac{2712}{168}$$

$$F_{crit} \Rightarrow df(1, 16) = 4.49$$

b) The main effect of A is not significant because the F-ratio is less than the critical value of 4.49.